GRADE -9

LESSON – 12, [HERON'S FORMULA]

Objective Type Questions

I - Multiple choice questions

1. A triangular colorful scenery is made in a wall with sides 50cm, 50 cm and 80cm, A golden thread is to hang from the vertex so as to just reach the side 80cm. How much length of golden thread is required?

a) 40cm b) 80cm c) 50cm d) 30cm

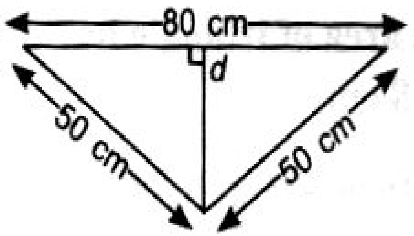
Sol. (d) Here a = 50cm, b=50cm, c = 80cm

S = $\frac{a+b+c}{2}$ = $\frac{50+50+80}{2}$ = $\frac{180}{2}$ = $90cm$

Area of triangle

= $\sqrt{90\,(90-50)(90-50)(90-80)}$

= $\sqrt{90 \times 40 \times 40 \times 10}$ = $1200\ cm^2$



$\Rightarrow$ Area of triangle = $\frac{1}{2} \times base \times height$

$\Rightarrow$ $1200 = \frac{1}{2} \times 80 \times d$

$\Rightarrow$ $d = \frac{1200}{40} = 30cm$

∴ Length of golden thread = 30cm

2. Given three sticks of lengths 10cm, 5cm and 3cm. A triangle is formed using the sticks, then area of the triangle will be

a) 50 cm^2 b) 25 cm^2

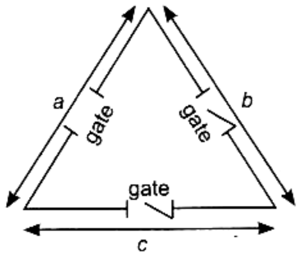
c) 15 cm^2 d) unable to form a triangle, so no area can be calculated.

Sol. (d) It is not possible to form a triangle because sum of two sides is less than third side (5 + 3 <10)

3. A gardener has to put double fence all round a triangular path with sides 120m, 80m and 60m. In the middle of each sides, there is a gate of width 10m. Find the length of the wire needed for the fence.

a) 250 m b) 490m c) 230m d) 460m

Sol. (d)



Sides of triangular path are

a=120m, b=80m, c=60m

Length of wire needed

= 2[perimeter of path – 3 (length of gate)]

= 2[120 + 80 + 60 – 3(10)] = 2[230] = 460m

4. A traffic signal board is board is equilateral in shape, with words 'SCHOOL AHEAD' with on it. The perimeter of the board is 180cm, then the area of the signal board is

a) 2826 cm^2 b) 1413 cm^2 c) 900 $\sqrt{3}cm^2$ d) 100 $\sqrt{3}cm^2$

Sol. (c), side of equilateral Δ

$= \frac{perimeter\ of\ equilateral\ A}{3}$

$= \frac{180}{3} = 60cm$

∴ Area of equilateral Δ = $\frac{\sqrt{3}}{4}(60)^2 = 900\sqrt{3}cm^2$

5. In a family with two sons, a father has a field in the form of a right angled triangle with sides 18m and 40m. He wants to give independent charge to his sons, so he divided the field in the ratio 2 : 1 : 1, the bigger part he kept for himself and divided remaining equally among the sons, find the total area distributed to the sons.

a) 360 m^2 b) 90 m^2 c) 180 m^2 d) 200 m^2

Sol. (c). Farmer divides the field in the ration 2 : 1 : 1,

∴ Area of right angular field

$= \frac{1}{2} \times base \times altitude$

$= \frac{1}{2} \times 18 \times 40 = 360m^2$

Area distributed to each son = $\frac{1}{4}[360] = 90m^2$

Total area distributed to sons = 2(90) = $180m^2$

6. The base of a right triangle is 8cm a hypotenuse is 10cm. Its area will be [NCERT Exemplar]

a) 24 cm^2 b) 40 cm^2 c) 48 cm^2 d) 80 cm^2

Sol. (a), Altitude of right triangle = $\sqrt{(10)^2 - (8)^2} = 6$

Area of right triangle = $\frac{1}{2} \times 8 \times 6 = 24\ cm^2$

7. Two sides of a triangle are 8cm and 11cm a perimeter of triangle is 32cm. Then value of 's' is

a) 19cm b) 20cm c) 21.5cm d) 16cm

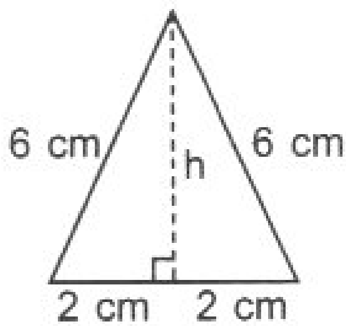
Sol. (d), Perimeter (2s) = 32cm ⟹ s = 16cm

8. The area of an isosceles triangle having base 4 cm and length of one of equal sides as 6cm

(a) $8\sqrt{2}cm^2$ (b) $16\sqrt{2}cm^2$ (c) $4\sqrt{2}cm^2$ (d) $16cm^2$

Sol. (a), h = $\sqrt{36-4} = \sqrt{32}$cm = 4 $\sqrt{2}$cm

∴ Area = $\frac{1}{2}$ x 4 x $4\sqrt{2}cm^2 = 8\sqrt{2}cm^2$



9. Area of a triangle with base 4cm and height 6cm is 12cm. State true or false. Justify your answer.

Sol. False, As area is in square units

10. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to the base is 3 : 2, then area of the triangle is __________.

Sol. Perimeter, 3x + 3x + 2x = 32 ⟹ x = 4cm

∴ Sides are 12cm, 12cm, 8cm

∴ Height = $\sqrt{(12)^2 - (4)^2}cm$ = $\sqrt{128}cm$ = 8 $\sqrt{2}$ cm

∴ Area of the triangle = $\frac{1}{2}$x 8 x 8$\sqrt{2}cm^2$ = 32$\sqrt{2}cm^2$

11. Find the area of triangle having base 6 cm and altitude 8 cm. [CBSE 2011]

Sol. Area of triangle = $\frac{1}{2}$x base x altitude

$= \frac{1}{2} \times 6 \times 8 = 24cm^2$

12. Find the area of triangle whose sides are 13cm, 14cm and 15cm.

Sol. Given a = 13cm, b = 14cm, and c = 15cm

Semi perimeter, s = $\frac{a+b+c}{2} = \frac{13+14+15}{2} = \frac{42}{2}$

= 21 cm

Using Heron's formula.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

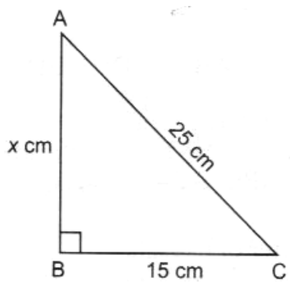
= $\sqrt{21(21-13)(21-14)(21-15)}$

= $\sqrt{21 \times 8 \times 7 \times 6}$ = $\sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3}$

= $\sqrt{\underline{7 \times 7} \times \underline{3 \times 3} \times \underline{2 \times 2} \times \underline{2 \times 2}}$

= 7 X 3 X 2 X 2 = 84 cm^2

13. If the base of a right – angled triangle is 15cm and its hypotenuse is 25cm, then find its area.



Sol. Using Pythagoras theorem in right – angled ΔABC we have

$AC^2 = AB^2 + BC^2$

⟹ $25^2 = x^2 + 15^2$

$\Rightarrow \quad x^2 = 25^2 - 15^2 = 625 - 225 = 400$

$\therefore \; x = \sqrt{400} = 20cm$

$\therefore$ Area of right – angled triangle = $\frac{1}{2} \times base \times height$

ar (ABC) = $\frac{1}{2} \times 15 \times 20 = 150 \; cm^2$

14. Two sides of a triangle are 13cm and 14cm and its semi-perimeter is 18cm. Find the third side of this triangle.

Sol. Semi – perimeter of triangle, s = $\frac{a+b+c}{2}$

$\Rightarrow \quad 18 = \dfrac{13 + 14 + c}{2}$

$\Rightarrow \quad c = 36 - 27 = 9$

$\therefore$ Third side of the triangle is 9cm

15. Find the area of an equilateral triangle with side 2 $\sqrt{3}$ cm

Sol. Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (side)^2$

$= \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2$ (Given side = $2\sqrt{3}$ cm)

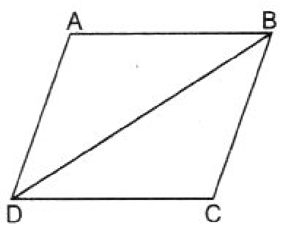
$= \frac{\sqrt{3}}{4} \times 4 \times 3 = 3\sqrt{3} cm^2$

II - Multiple choice questions

1. A rhombus shaped field has green grass for 36 cows to graze. If each side of the field is 30m and longer diagonal is 48m, then how much area of grass each cow will get, if $216m^2$ of area is not to be grazed.

a) $6m^2$ b) $12m^2$ c) $18m^2$ d) $29m^2$

Sol. (c) ABCD is a rhombus shaped field side = 30m, diagonal BD = 48m



So, a = 30m, b = 30m, c=48m

s = $\frac{a+b+c}{2}$ = $\frac{30+30+48}{2}$ = $54m$

Area of ABD = $\sqrt{54 \times 24 \times 24 \times 6}$

= 24 × 3 × 6 = 432m^2

Area of rhombus = 2 x area of ΔABD

= 2 x 432 = 864m^2

Area to be grazed = 864 – 216 = 648m^2

Area grazed by each cow = $\frac{648}{36}$ = $18m^2$

2. The area of a regular hexagon 'a' is the sum of the areas of the five equilateral triangles with side 'a'. Write true or false and justify your answer [NCERT Exemplar]

Sol. False, because regular hexagon consists of six equilateral triangles of side 'a'

I - Short answer type questions

1. Find the area of an isosceles triangle whose one side is 10cm greater than each of its equal sides and perimeter is 100cm [CBSE 2014]

Sol. Let equal sides of an isosceles triangle be x cm. Therefore, the length of its greater side = $(x + 10)$cm

$\Rightarrow$ $x + x + (x + 10) = 100$

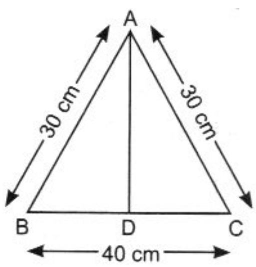
$\Rightarrow$ $3x = 100 - 10 = 90$

$\Rightarrow$ $x = \frac{90}{3} = 30\ cm$

∴ Base of an isosceles triangle = 10 + x = 10 + 30 = 40 cm

We know that in an isosceles triangle, its altitude bisects the base.

∴ ΔABC is a right – angled triangle.



$AB^2 = BD^2 + AD^2$ [Using Pythagoras Theorem]

$\Rightarrow$ $30^2 = 20^2 + AD^2$

($\because$ BD = $\frac{1}{2} BC = \frac{1}{2} \times 40 = 20cm$)

$\Rightarrow$ $AD^2 = 30^2 - 20^2 = 900 - 400 = 500$

$\Rightarrow$ $AD = \sqrt{500} = 10\sqrt{5}\,cm$

$\therefore$ Area of an isosceles triangle = $\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 40 \times 10\sqrt{5} = 200\sqrt{5}cm^2$

2. The sides of triangle are 8cm, 15cm, 17cm, Find the area. [CBSE 2016]

Sol. Given a = 8cm, b = 15cm and c = 17cm

The semi-perimeter of triangle,

$$s = \frac{a+b+c}{2}$$

$\Rightarrow$ $s = \frac{8 + 15 + 17}{2} = \frac{40}{2} = 20cm$

Using Heron's formula.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{20(20-8)(20-15)(20-17)}$

$= \sqrt{20 \times 12 \times 5 \times 3} = \sqrt{5 \times 4 \times 4 \times 3 \times 5 \times 3}$

$= \sqrt{\underline{5 \times 5} \times \underline{4 \times 4} \times \underline{3 \times 3}} = 5 \times 4 \times 3 = 60cm^2$

3. Find the perimeter of an isosceles right-angled triangle having an area of $5000m^2$ (Use $\sqrt{2} = 1.41$) [CBSE 2015]

Sol. Let ΔABC be an isosceles right-angled triangle in which AB = BC = x m.

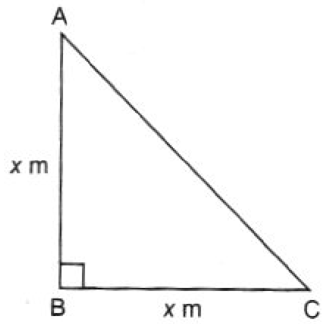
$\therefore$ Area of ΔABC = $\frac{1}{2} \times base \times height$

$5000 = \frac{1}{2} \times x \times x$ v [Given area = $5000m^2$]

$\Rightarrow$ $x^2 = 5000 \times 2 = 10000$

$\Rightarrow$ $x = \sqrt{10000} = 100\,m$

Using Pythagoras theorem in an isosceles right-angled ΔABC, we have



$AC^2 = AB^2 + BC^2 = x^2 + x^2 = 2x^2$

$\therefore$ AC = $x\sqrt{2} = 100\sqrt{2}\,m$

= 100 x 1.41 = 141m

$\therefore$ Perimeter of an isosceles right – angled ΔABC = sum of all sides

= AB+ BC + AC

= 100 + 100 + 141 = 341m

4. The sides of triangle are 100m 120m and 140m, Find its area. (Use $\sqrt{6}$ = 2.45) **[CBSE2016]**

Sol. Given a = 100m, b = 120m, c = 140m

$\therefore$ Perimeter of triangle, 2s = a + b + c

$\Rightarrow \quad 2s = 100 + 120 + 140 = 360m$

$\Rightarrow \quad Semi\ perimeter, s = \frac{360}{2} = 180m$

Using Heron's formula

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{180(180-100)(180-120)(180-140)}$

= $\sqrt{180 \times 80 \times 60 \times 40} = \sqrt{60 \times 3 \times 40 \times 2 \times 60 \times 40}$

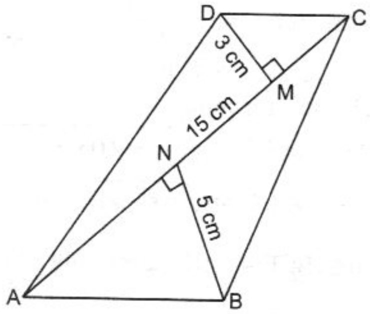
= $\sqrt{\underline{60 \times 60} \times \underline{40 \times 40} \times 6}$

= 60 x 40 x $\sqrt{6}$ = 2400 x 2.45 = 5880m^2

II – Short Answer Type Question

1. Find the area of quadrilateral ABCD as shown in the figure:

Sol. Area of quadrilateral ABCD = ar (ΔABC) + ar (ΔADC)



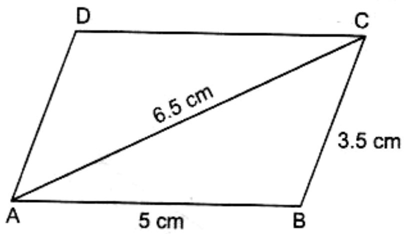
$= \frac{1}{2} \times AC \times BN + \frac{1}{2} \times AC \times DM$

$= \frac{1}{2} \times 15 \times 5 + \frac{1}{2} \times 15 \times 3$

$= \frac{15}{2} \times (5 + 3) = \frac{15}{2} \times 8 = 60cm^2$

2. Two adjacent sides of a parallelogram measures 5cm and 3.5cm. One of its diagonal measures 6.5cm. Find the area of the parallelogram.

Sol. Let ABCD be the parallelogram with AB = 5cm. BC = 3.5cm and AC = 6.5cm as shown in figure.



Semi – perimeter of ΔABC

$s = \frac{a+b+c}{2}$

$= \frac{5 + 3.5 + 6.5}{2} = \frac{15}{2} = 7.5cm$

Using Heron's formula.

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7.5(7.5-5)(7.5-3.5)(7.5-6.5)}$

$= \sqrt{7.5 \times 2.5 \times 4 \times 1}$

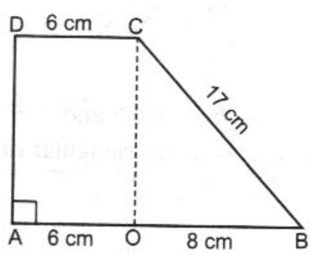
$= \sqrt{7.5 \times 10} = \sqrt{75} = 5\sqrt{3}cm^2$

We know that the diagonal of a parallelogram divides it into two congruent triangles of equal area.

∴ Area of parallelogram ABCD = 2 x ar(ΔABC)

= 2 X 5 $\sqrt{3}$ = 10 $\sqrt{3}$ cm^2

3. Calculate the area of trapezium as shown in the figure [CBSE 2015]



Sol. In ΔBOC, ∠O = 90^0 as CO || AD

Using Pythagoras theorem in right – angled ΔBOC, we have

$BC^2 = OC^2 + OB^2$

$\Rightarrow 17^2 = OC^2 + 8^2$

$\Rightarrow OC = \sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225}$

∴ OC = 15cm

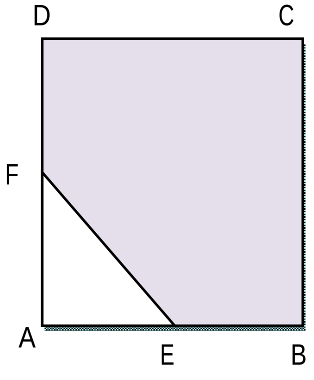
∴ $Area$ of trapezium ABCD = $\frac{1}{2} \times (AB + CD) \times OC$

$= \frac{1}{2} \times (14 + 6) \times 15$

$(\because AB = AO + OB = 6 + 8 = 14cm)$

$= \frac{1}{2} \times 20 \times 15 = 150cm^2$

4. In the given figure, ABCD is a square of side 4cm, E and F are mid-points of AB and AD respectively. Find the area of the shaded region. [CBSE 2016]



Sol. Area of square ABCD = $(side)^2$

$(\because side\ of\ square = 4cm)$

= 4^2 = 16cm^2

Area of right – angled ΔEAF = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times AE \times AF$

= $\frac{1}{2} \times 2 \times 2$ $(\because AE = \frac{1}{2}AB\ and\ AF = \frac{1}{2}AD)$

= 2cm^2

$\therefore$ *Area* of shaded region = Area of square ABCD - Area of ΔEAF = 16 – 2 = 14cm^2

5. Find the area of the parallelogram, whose one diagonal is 6.8cm and the perpendicular distance from the opposite vertex is 7.5cm [HOTS]

Sol. We know that the diagonal of parallelogram divides it into two congruent triangles of equal area.

$\therefore$ Area of one triangle = $\frac{1}{2} \times base \times height$

= $\frac{1}{2} \times 6.8 \times 7.5$

= 25.5 cm^2

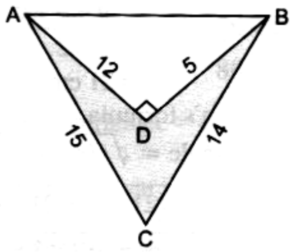
$\therefore$ *Area of parallelogram*

= 2 x Area of one congruent triangle

= 2 x 25.5 = 51cm^2

III – Short Answer Type Question

1. **Find the area of shaded region in the given figure.**
[All measurements are in cm] [CBSE 2014]



Sol. Area of right – angled ΔADB

$= \frac{1}{2} \times base \times height = \frac{1}{2} \times BD \times AD$ [Base = BD height = AD]

$= \frac{1}{2} \times 5 \times 12 = 30cm^2$

Using Pythagoras theorem in an isosceles right-angled ΔADB, we have

$AB^2 = AD^2 + BD^2$

$= 12^2 + 5^2 = 144 + 25 = 169$

∴ Perimeter of triangle,

2s = AB + BC + AC

= 13 + 14 + 15 = 42cm

$Semi\ perimeter, s = \frac{42}{2} = 21cm$

Using Heron's formula

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-15)(21-14)}$

$= \sqrt{21 \times 8 \times 6 \times 7} = \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7}$

$= \sqrt{\underline{7 \times 7} \times \underline{3 \times 3} \times \underline{2 \times 2} \times \underline{2 \times 2}}$

= 7 x 3 x 2 x 2 = $84cm^2$

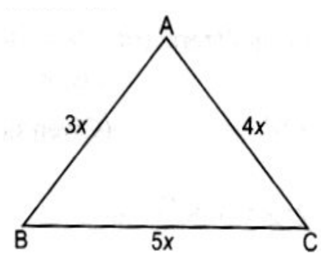
Area of shaded portion = ar (ΔABC) – ar (ΔADB)

= 84 – 30 = $54cm^2$

2. The perimeter of a triangular garden is 900cm and its sides are in the ration 3 : 5 : 4. Using Heron's formula, find the area of triangular garden. [CBSE 2015]

Sol. Suppose that the sides of a triangular garden (in cm) are 3x, 5x, and 4x

Perimeter of triangular garden = 900cm



⟹ 900 = 3x + 5x + 4x

⟹ 900 = 12x

⟹ $x = \frac{900}{12} = 75cm$

So, the sides of triangular garden are 3 x 75cm, 5 x 75cm and 4 x 75cm, i.e.225cm, 375cm and 300cm

Now we have semi-perimeter,

s = $\frac{225+375+300}{2}$ cm = $\frac{900}{2}$ = 450cm

Using Heron's formula

Area of triangular garden = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{450(450-225)(450-375)(450-300)}$

= $\sqrt{450 \times 225 \times 75 \times 150}$

= $\sqrt{225 \times 2 \times 225 \times 75 \times 75 \times 2}$

= $\sqrt{\underline{225 \times 225} \times \underline{75 \times 75} \times \underline{2 \times 2}}$

= 225 x 75 x 2 = 33,750cm^2 = 3.375m^2

3. Find the area of triangle whose perimeter is 180cm and its two sides are 80cm and 18cm. Calculate the altitude of triangle corresponding to its shortest side

[CBSE 2015]

Sol. Given a = 80cm and b = 18cm

Perimeter of triangle = a + b + c

⟹180 = 80 + 18 + c

∴ c = 180 – 98 = 82cm

and semi – perimeter, s = $\frac{perimeter}{2}$ = $\frac{180}{2}$

= 90cm

Using Heron's formula

Area of triangular = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{90(90-80)(90-18)(90-82)}$

= $\sqrt{90 \times 10 \times 72 \times 8}$

= $\sqrt{10 \times 9 \times 10 \times 9 \times 8 \times 8}$

= $\sqrt{\underline{10 \times 10} \times \underline{9 \times 9} \times \underline{8 \times 8}}$

= 10 x 9 x 8 = 720cm^2

The shortest side of triangle = 18cm

∴ Area of $triangle = \frac{1}{2} \times base \times altitude$

$720 = \frac{1}{2} \times 18 \times h$

∴ $h = \frac{720}{9} = 80\ cm$

∴ Altitude of triangle corresponding to its shortest side (18cm) is 80cm

IV – Short Answer Type Question

1. A floor design is made on a floor of a room by joining four triangular tiles of dimensions 12cm, 20cm and 24cm, each. Find the cost of the tiles at the rate of Rs.$\sqrt{14}$per cm^2 [HOTS]

Sol. Given a = 12cm, b = 20cm, c = 24cm

∴ Semi – perimeter, $s = \frac{a+b+c}{2}$

$$s = \frac{12 + 20 + 24}{2} = \frac{56}{2} = 28cm$$

Using Heron's formula.

Area of one triangular tile = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{28(28-12)(28-20)(28-24)}$

$= \sqrt{28 \times 16 \times 8 \times 4}$

$= \sqrt{14 \times 2 \times 4 \times 4 \times 4 \times 2 \times 4}$

$= \sqrt{\underline{4 \times 4} \times \underline{4 \times 4} \times \underline{2 \times 2} \times 14}$

$= 4 \times 4 \times 2 \times \sqrt{14} = 32\sqrt{14} cm^2$

∴ Area of 4 such tiles = $4 \times 32\sqrt{14}$

$= 128\sqrt{14}\ cm^2$

Cost of one tile = Rs. $\sqrt{14}\ per\ cm^2$

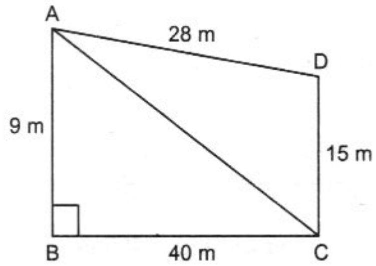
∴ Total cost of 4 tiles = Rs.$\sqrt{14} \times 128\sqrt{14}$

= Rs 128 x 14 = 1792

Hence cost of designing the floor = Rs.1792

2. A forest reservoir is in the shape of quadrilateral whose sides taken in order are 9m, 40m, 15m and 28m. If the angle between first two sides is a right angle, find the area of a forest reservoir. [HOTS]

Sol. ΔABC is a right-angled triangle. So, by using Pythagoras theorem in right – angled ΔABC, we have



$AC^2 = AB^2 + BC^2$

$= 9^2 + 40^2$

= 81 + 1600 = 1681

∴ AC = $\sqrt{1681}$ = 41m

∴ Area of ΔADB = $\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 40 \times 9 = 180cm^2$

Now, in ΔACD

Let a = AC = 41m, b = CD = 15m, and c = AD = 28m

Semi-perimeter, s $= \frac{a+b+c}{2} = \frac{41+15+28}{2} = \frac{84}{2} = 42cm$

Using Heron's formula.

Area of ΔACD = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-41)(42-15)(42-28)}$

$= \sqrt{42 \times 1 \times 27 \times 14}$

$= \sqrt{42 \times 1 \times 27 \times 14}$

$= \sqrt{42 \times 3 \times 3 \times 3 \times 3 \times 14}$

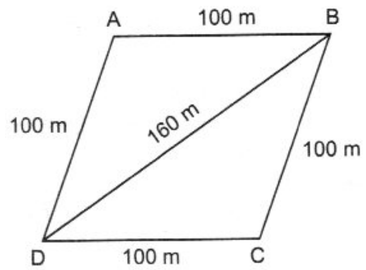
$= \sqrt{\underline{14 \times 14} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

$= 14 \times 3 \times 3 = 126 \ m^2$

∴ Area of quadrilateral ABCD =ar (ΔABC) + ar(ΔACD) = 180 + 126 = $306m^2$

3. Sanya has a piece of land which is in the shape of a rhombus. She wnts her one daughter and one son to work on the land and produce different crops to suffice the needs of their family. She divided the land in two equal parts. If the perimeter of the land is 400m and one of the diagonal is 160m, how much area each of them will get?

Sol. Let ABCD be the field.



Perimeter of field = 400m

So, each side = 400m ÷ 4 = 100m

i.e., AB = AD = 100 m

Let diagonal BD = 160 m

Then semi-perimeter of ΔABD is given by

s $=\frac{100+100+168}{2} = 180m$

Using Heron's formula.

Area of ΔABD = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{180(180-100)(180-100)(180-160)}$

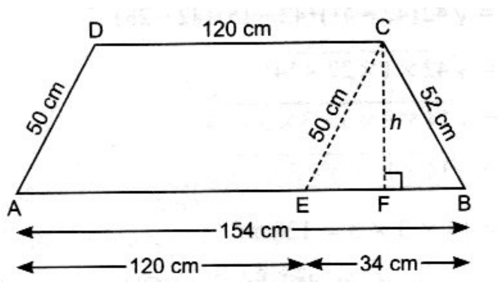
= $\sqrt{180 \times 80 \times 80 \times 20}m^2$ = 4800m^2 .

Therefore, each of them will get an area of 4800m^2 since ar(ΔABD) = ar(ΔBCD)

4. Two parallel sides of a trapezium are 120cm and 154cm and other sides are 50cm and 52cm. Find the area of trapezium [CBSE 2011]

Sol. Let ABCD be a trapezium in which AB = 154cm, CD = 120cm, AD = 50cm, BC=52cm

Construction: Draw CE||AD and CF ⊥ AB



Now, CD || AB and CE || DA

∴ AECD is a parallelogram.

⟹ CE = AD = 50 cm

and CD = AE = 120cm

∴ $BE = AB - AE = 154 - 120 = 34cm$

In ΔBEC, semi-perimeter,

s =$\frac{a+b+c}{2}$

$$\Rightarrow s = \frac{50 + 52 + 34}{2} = \frac{136}{2} = 68cm$$

Using Heron's formula.

Area of ΔCEB = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{68(68-50)(68-52)(68-34)}$

= $\sqrt{68 \times 18 \times 16 \times 34}$

= $\sqrt{34 \times 2 \times 2 \times 9 \times 4 \times 4 \times 34}$

= $\sqrt{\underline{34 \times 34} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{4 \times 4}}$

= 34 × 2 × 3 × 4

= 34 × 24 = 816cm^2

But Area of ΔCEB = $\frac{1}{2} \times base \times height$

⟹ 816 = $\frac{1}{2} \times BE \times CF$

816 = $\frac{1}{2} \times 34 \times CF$

⟹ CF = $\frac{816}{17}$ = 48cm

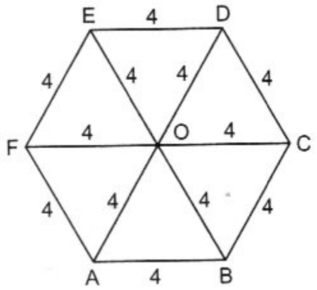
∴ $Area$ of trapezium

ABCD = $\frac{1}{2}$ × $(Sum\ of\ parallel\ sides) \times height$ = $\frac{1}{2}$ × $(AB + CD) \times CF$

= $\frac{1}{2}$ × (154 + 120) × 48 = $\frac{1}{2}$ × 274 X 48 = 6576cm^2

5. Find the area of a regular hexagon whose one side is 4 units

Sol. The six sides of regular hexagon are of equal length. The point of intersection of its diagonal divides it into six equilateral triangles each of side 4 units as shown in figure.



Area of an equilateral triangle each side of 'a' unit = $\frac{\sqrt{3}}{4}$ × a^2

Area of ΔOAB = $\frac{\sqrt{3}}{4}$ × 4^2 = $4\sqrt{3}$ sq. units

Area of hexagon ABCDEF = 6 x ar(ΔOAB)

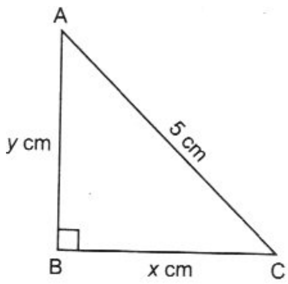
= 6 X 4 $\sqrt{3}$

= 24 $\sqrt{3}\, sq.units$

I – Long Answer Type Question

1. The perimeter of a right – angled triangle is 12 cm and its hypotenuse is of length 5cm. Find the other two sides and calculate its area. Verify the result using Heron's formula.

Sol. Let ΔABC be the right – angled triangle.



Suppose AB = y cm and BC = xcm

Now, $x + y + 5 = 12$ [Given perimeter of triangle is 12cm]

$\Rightarrow$ $x + y = 7$ ……… (i)

Using Pythagoras theorem in right-angled ΔABC, we get

$x^2 + y^2 = 25$ …….. (ii)

Squaring (i) both sides, we get $(x + y)^2 = 7^2$

$\Rightarrow$ $x^2 + y^2 + 2xy = 49$

$\Rightarrow$ $25 + 2xy = 49$

$\Rightarrow$ $xy = 12$ ………. (iii)

∴Area of right – angled ΔABC = $\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times x \times y = = \frac{1}{2} \times 12$

$= 6cm^2$

Now, consider $(x - y)^2 = (x + y)^2 - 4xy$

$= 49 - 4 \times 12 = 1$

$\Rightarrow$ $x - y = \pm 1$

If $x + y = 7$ and $x - y = 1$, we get $x = 4, y = 3$

If $x + y = 7$ and $x - y = -1$, we get $x = 3, y = 4$

Therefore, length of the other two sides of triangle are 3cm and 4cm

Verification of Area of ΔABC by Heron's formula:

semi–perimeter, s = $\frac{3+4+5}{2} = 6cm$

Using Heron's formula,

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6 \times 3 \times 2 \times 1}$

$= \sqrt{6 \times 6} = 6cm^2$

Which is same as obtained earlier.

Hence result is verified.

2. How much area of triangle will increase in percentage, if each side of the triangle is doubled? [HOTS]

Sol. Let a, b, c be the sides of the triangle

Let its semi-perimeter be s_1

∴ $s_1 = \frac{a+b+c}{2}$

Using Heron's formula

Area of $triangle A_1 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad A_1 = \sqrt{s_1(s_1-a)(s_1-b)(s_1-c)}$

When the sides of triangle are doubled, i.e., 2a, 2b, 2c, then its

Semi-perimeter,

$s_2 = \frac{2a+2b+2c}{2} = 2\left(\frac{a+b+c}{2}\right) = 2s_1$

Using Heron's formula

Area of $triangle A_2 = \sqrt{s_2(s_2-2a)(s_2-2b)(s_2-2c)}$

$\Rightarrow \quad A_2 = \sqrt{2s_1(2s_1-2a)(2s_1-2b)(2s_1-2c)}$ $(\because s_2 = 2s_1)$

$= 4\sqrt{s_1(s_1-a)(s_1-b)(s_1-c)}$

$\Rightarrow A_2 = 4\,A_1 = 4$ x Area of original triangle

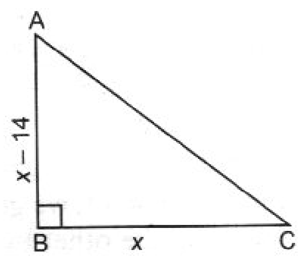
Hence, percentage increase in area

$= \frac{Increase\ in\ area}{Original\ area} \times 100 = \frac{A_2 - A_1}{A_1}$ x 100

$= \left(\frac{A_2}{A_1} - 1\right)$ x 100 = (4 – 1) × 100 = 300%

3. The difference between the two adjoining sides containing right angle of a right-angled triangle is 14cm. The area of triangle is 120 cm^2. Verify this area by using Heron's formula [HOTS]

Sol. Let ΔABC be the right angled triangle with ∠B = 90^0



Let BC = x cm and AB = (x-14)cm

Area of right – angled ΔABC = $\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times BC \times AB$

$\Rightarrow \quad 120 = \frac{1}{2} \times x \times (x - 14)$

$\Rightarrow \quad 240 = x^2 - 14x$

$\Rightarrow \quad x^2 - 14 - 240 = 0$

$\Rightarrow \quad x^2 - 24x + 10x - 240 = 0$

$\Rightarrow \quad x(x - 24) + 10(x - 24) = 0$

$\Rightarrow \quad (x - 24)(x + 10) = 0$

$\Rightarrow \quad either\ x - 24 = 0\ or\ x + 10 = 0$

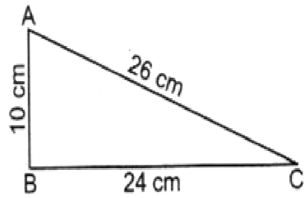
$\Rightarrow \quad either\ x = 24\ or\ x = -10$

Since length cannot be negative, so by ignoring x = -10, we get

BC = x = 24cm and AB = x – 14 = 24 – 14 = 10cm

And hypotenuse, AC = $\sqrt{AB^2 + BC^2} = \sqrt{10^2 + 24^2}$

$= \sqrt{100 + 576} = \sqrt{676} = 26$cm



Verification of Area of ∆ABC by Heron's formula

Let a = 24cm, b = 10cm, c = 26cm

∴ Semi-perimeter,

s = $\frac{a+b+c}{2} = \frac{24+10+26}{2} = \frac{60}{2} = 30cm$

Using Heron's formula

Area of ∆ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{30(30-24)(30-10)(30-26)}$

$= \sqrt{30 \times 6 \times 20 \times 4} = \sqrt{6 \times 5 \times 6 \times 5 \times 4 \times 4}$

$= \sqrt{\underline{6 \times 6} \times 5 \times 5 \times 4 \times 4}$

= 6 x 5 x 4 = 120cm^2

Hence result is verified.